# What if science and mathematics education was based on historical dialectical materialism?

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In the film, "Everyone for Themselves and God Against All" by Werner Herzog, there is a Professor who asks the protagonist, Kaspar, to pretend that there is a village in which all the people only tell the truth, and another village in which people only tell lies (Herzog, 1974a). He asks Kaspar how he could know which of these two villages a man comes from by asking the man just one question. Without pausing, the Professor gave the the question which answers this problem of logic:

"'If you came from the other village, would you answer no if I were to ask you whether you came from the liars' village?' By means of a double negative, the liar is forced to tell the truth. This construction forces him to reveal his identity, you see. That's what I call logic via argument to the truth!"

https://www.youtube.com/watch?v=C9uqPeIYMik

Although the Professor insisted that this is the only possible question which could solve the problem, Kaspar had another answer - he said that he would ask the man whether he was a tree-frog, and from his answer it would be clear whether he was lying or not. But the Professor replied:

"No, that's not a proper question. That won't do, I can't accept it as a question. That's not logic. Logic is deduction, not description. What you've done is describe something, not deduce it."

A woman who was also sitting at the table said: "But I understood his question." But the Professor said:

"Understanding is secondary. The reasoning is the thing. In logic and mathematics, we do not understand things... We reason and deduce. I cannot accept that question."

This dialogue provides us with an example in which a common belief about the nature of mathematics is highlighted. In mathematics, what is right and what is wrong is generally considered as being just a matter of logic. Mathematics is not concerned about understanding physical reality (things, matter, or processes). It is a form of idealism, by which we mean that ideas, reasoning, and logic are basic, and can exist independently of physical reality.

But in this paper we will question this conception of the nature of mathematics. We will question whether mathematics is really so different from science. In the above dialogue, the reason that the Professor did not accept Kaspar's question about the tree-frog, is that the way someone would know whether the man was a tree-frog depends on observing physical reality, not just on logical deduction. Perhaps it is more science than mathematics. The Professor did not not consider observation of physical reality to be connected to mathematics. While mathematics could accordingly be considered as idealist, clearly science is based on observing physical reality. Thus, science must be materialistic. In science, the truth of an idea, statement, theory, or explanation can be ascertained through direct or indirect observations of physical reality - and the logic may be probabilistic, inductive, and very difficult to prove (not that the logic of mathematics is always very simple).

However, we wonder whether mathematics needs to be considered to be so different from science. Can mathematics and mathematics education also be materialist in some sense? Or rather, Is mathematics really idealist? Are the mathematical principles really the starting point of any authentic engagement in mathematics? What if a particular mathematical model/conceptual structure does not properly/accurately describe some phenomena in nature or history? What would be more reasonable - a different (mathematical) model or a different phenomenon which fits into the model that we consider fundamental? These questions need to be taken into consideration by those of us who design curricula, textbooks, and teaching/learning materials and methods.

The above dialogue also raises another problem. Suppose we consider another way to find out which village the man comes from: we could ask him what might sound like a purely logical mathematical question, "Is 1+1=2?" Again, the professor would say that it would not be an acceptable question, since he had said that there is only one solution to this problem. This could imply that he would not consider the truth of 1+1=2 to be valid in this context because it cannot be deduced just from the statements given in the problem.

However, if "1+1=2" is a universal statement, implying that its truth is independent of any context, its universal property would imply that its truth could be arrived at even in this context.

Actually, mathematical ideas (concepts/models) such as integers, addition/subtraction, multiplication/division do need a context from which they can be developed, learnt and taught. And, their definitions can be, and in fact are formalized later on to mask their material origins. And any new mathematical ideas that arise later do indeed emerge from people interacting with the material world, who later on try to formalize their new ideas.

Another problem which is also raised in the same Werner Herzog film, is that science is sometimes taught as if it is idealist rather than materialist. This is shown in the following dialogue (Herzog, 1974b):

Teacher: [referring to the apples he is carrying in his arms] Look, these are last year's apples. Big and red, aren't they? And those apples on the tree will look just the same soon.

> Kaspar: How do they do that?
>
> Teacher: Time does it, Kaspar.
>
> Priest: And the Lord's plan.
>
> [a couple of apples the Teacher is carrying fall to the ground and he bends down to pick them up.]
>
> Kaspar: Let the apples lie, they're tired and they want to sleep.

Teacher: Kaspar, an apple can't be tired. Apples don't have lives of their own, they follow our will. I'm going to roll one down the path, it'll stop where I want it to.

[He rolled it, but the apple bounced off the path.]

Kaspar: The apple didn't stop, it hid in the grass.

... [Then the teacher gets the priest to hold out his foot to stop another apple which he rolls, but instead of stopping where he wants it to, it bounces over the foot.]

Kaspar: Smart apple! It jumped over his foot and ran away!

https://www.youtube.com/watch?v=x7hGx9okkE4

This scene describes a common (idealist) approach towards science education: teachers try to fit observations within a conceptual structure and keep trying to get an experiment to produce results which fit into their particular, predetermined concept or theory. According to this approach, the concepts (ideas) are taken to be more fundamental than what actually happens in physical reality.

This contrasts with a materialist approach in which the observations would lead to concepts or theories which may be different from what was hypothesized or even different from what was previously proven. The 'stuff' is the authority, as Eleanor Duckworth says (Duckworth, 2012).

In this paper we will develop a historical dialectical materialist (HDM) framework and claim that both mathematics and science education should be based on an HDM ideology. We will base our arguments mainly on two sets of teaching/learning experiences, in which we compare and contrast teaching methods which are more idealist with those that are more HDM. In one set of experiences we contrast the way even and odd numbers are introduced in the Class VI NCERT mathematics textbook to an experience which occurred when one of us was teaching a session on even and odd numbers at a school for the blind (D'Souza, 2016). In the other set of experiences, we contrast three different ways that we have previously taught topics related to tree identification at the middle school level (the third is described in Singh, Shaikh & Haydock, (2016)).

Based on these comparisons, we will make some concrete suggestions on how teaching science and mathematics can be improved by explicitly basing it on an HDM ideology.

## Some examples and their ideologies

**Example A**

The NCERT Class VI Mathematics textbook introduces even and odd numbers as follows:

Do you observe any pattern in the numbers 2, 4, 6, 8, 10, 12, 14, ...? You will find that each of them is a multiple of 2.

These are called *even numbers*. The rest of the numbers 1, 3, 5, 7, 9, 11, ... are called *odd numbers*.

This is followed by questions such as:

Fill in the blanks:

The smallest even number is _______.

**Example B**

With a group of 10 students (ages 11-19), seated in a circle on the floor, the teacher began a discussion by asking the students how to characterise numbers as odd or even. A number of students agreed that numbers like 2, 4, 6, ..., which could be evenly divided by 2 should be categorized as *even*, while numbers like 1, 3, 5,... that could not, would be odd. When the teacher asked about the number zero, the students said that zero is both odd and even because it leaves no remainder when divided by 2 (hence it is even), but we cannot divide zero by two since we have nothing to divide (hence, zero is odd). During further discussion, one student changed his mind, arguing that all natural numbers have the property that "odd + odd = even"; "even + even = even"; "odd + even = odd" and "even + odd = odd" and used this property to make his definition. Since 3-3=0, he concluded that zero is even, and not odd. When the teacher asked about the number -4 he stated that before deciding whether its odd or even, we need to know where did these numbers like -1, -2 come from. He said that there has to be a reason. He stated that for example when we have some number of ice creams which can be divided among two people, we call those numbers even. He then demanded a reason for having negative numbers only after which could it be decided whether they could be categorized as even or not.

When the teacher asked the students to think of contexts where negative numbers are necessary, he gave the example of the time he visited a mall in which the lift that displayed negative numbers to indicate the basements. Later on he said that negative numbers must be older than malls, and maybe during the Harappan civilization building structures which had some sort of basements could have given rise to the concept of negative numbers. The discussions continued with other hypotheses and examples that led the students to conclude that it makes more sense to categorize -2, -4, … as even numbers so that they fit into a continuous pattern of alternating even and odd numbers whether read backwards or forwards.

Some students could not understand the discussion, to which, a classmate would explain the ideas through a context that was familiar to them.

Later on the students again chose to deny zero as being an even number, since zero has a property different from other even numbers: "If you kept dividing an even number by 2, you'd reach an odd number. This does not work for zero." This contradicted the teacher's claim of zero being just another even number.

**Mathematics content versus process**

In Example A, students are asked whether they observe a pattern. It was taken for granted that only one pattern would be found. The process of "finding" is assumed to either, inevitably lead the student to the one correct idea, or get the teacher to ensure that the student "finds" the correct pattern. The students, thus, don't quite have the liberty of actually observing patterns since they are asked to find the one correct pattern which, in fact, is shown to them in the immediate following sentence. The definition is fixed and independent of the need to have a particular set of numbers demarcated and called even numbers. The underlying assumption is that, a concept has one fixed definition which is inevitably independent to the lives of people. Being a fixed definition, the history of a concept is thus irrelevant. The teaching of mathematics thus means the teaching of a set of concepts, procedures, algorithms, etc. and testing to check if the student has grasped that specific 'knowledge'. The learning objective is to get the student to "know" the body of knowledge. As it was assumed that there was a fixed body of knowledge, the students could be easily tested and ranked according to the amount of 'knowledge' they can recall in a format defined by an external authority. The teacher is thus conditioned to be authoritative.

In example B, the students explored and observed various patterns. And based on some of these patterns a student developed and defined a concept which he shared with the class. He would demand the history of the concept as he asserted it to be essential to produce a definition. Only after finding a rationale for needing a definition would the students accept and produce a definition. The definition of the concept was developed based on its properties which in turn was dialectically linked to its definition. The learning objective was the process of doing mathematics. Further, the mathematics discourse was interconnected with the experiences of students. Having less emphasis on the body of knowledge and more on the process, the teacher too was part of the learning process.

**Example C**

A teacher asked a class of 38 students to open their textbooks to a section which showed some pictures of various trees, with the common and scientific names and short descriptions of each tree. The text was read out loud by the teacher, and a few students whom she called on. The teacher wrote questions on the board and called on certain students to give the answers which they had found in the text. The teacher wrote them on the board for the students to copy into their notebooks and memorise. No students asked any questions or made any unsolicited comments, except in whispers to each other, which the teacher either ignored or admonished. At the end of the year the questions and answers were reviewed. Some of the same questions appeared in a final exam, and students were given full marks if their answers were the same as what they had been given. The final marks on the questions pertaining to tree identification showed a wide distribution, with 3-4 students 'topping' the class with near-perfect scores, and 3-4 students 'failing' with zero marks.

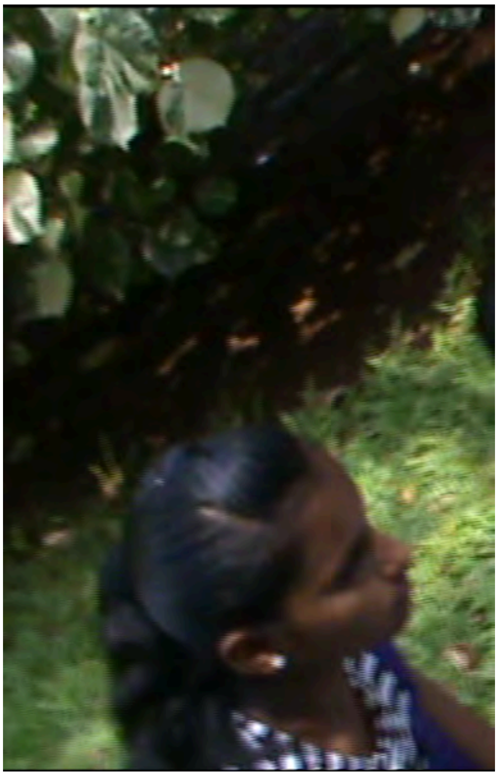
**Example D**

A teacher told a class of 24 middle school science students that they would be going outside to identify nearby trees. Before going out, the teacher told the class the main features by which trees are identified and how to use a guide book and a key (with a flow chart) to identify trees. The teacher divided the class into small groups, assigned group leaders, and gave them a copy of the guide book and a worksheet to fill up to so that they could follow a step-by-step procedure to identify each tree and record its name and identifying characteristics. The teacher announced that this would be a competition to see which group could identify the most trees in 20 minutes. The students went out, and after 20 minutes most groups managed to correctly fill up the worksheet showing that they had identified at least 5-6 trees. The students shared their work with each other and wrote the names of each of the trees next to their pictures. Afterwards, the students were tested in a written exam in which they were supposed to write the names of the trees next to their pictures, and there was a distribution of marks similar to those of Example A.

**Example E**

Some teachers noticed a variegated bhendi tree which they found interesting because some of its leaves were white. They wanted to find out whether students would find it interesting and whether they would discuss it and ask questions about how white leaves could survive, even if the teacher did not ask them to ask questions. Therefore the teachers asked a group of 11 middle school students to come to the part of the garden which contained the tree (but without mentioning the tree), and the teachers then kept quiet, observing and listening to the students. One group of students first walked past the bhendi tree to another tree which happened to have a label on it. After discussing it and asking each other a few questions about it, another group started discussing the names of other trees nearby. When they turned to the variegated bhendi tree, they asked each other what it was, but none of the students knew. However, they began to invent names, such as 'show [ornamental] tree', 'white-leaf-tree', 'multicolour-tree', etc. They then began to discuss various aspects of the tree, asking each other questions and trying to find answers by doing their own investigations. For example, they had discussions on what colours of leaves there were, whether the green colour was being worn off of some leaves, whether its peculiar buds were flowers, and whether the tree had thorns. After about 40 minutes at the tree, one girl noticed an unvariegated bhendi tree nearby, and asked a teacher if it was having the same name as the tree they had been investigating. The teacher confirmed this and thus this student finally did get to know the common name of the tree. Later this name was used in full-class discussions. These students were not formally tested but two years later, the teachers noticed that some but not all of the students remembered that ‘bhendi’ was the name of the tree.

The variegated bhendi tree



Srushti in front of the variegated bhendi tree, noticing the regular bhendi tree on the right.

**Science process vs science content**

Clearly, the degree of teacher-led guidance is greater in D than in E, and even greater in C. Example C is the most didactic and E the most activity-based. However, here we will focus on other aspects in order to show the differences that teaching based on an HDM ideology can make.

If the learning objective was that students learn to remember the names of trees, examples C and D could be considered to be fairly successful. Example E might be considered to be less successful, although some of the students did learn the name of at least one new tree. However, we think that what made E more worthwhile is that the learning objective in E was mainly to learn the process of science whereas in C (and D) the objective was to learn ‘science content’.

In C, the students were remembering the answers to the teacher's questions about the names and characteristics of trees, whereas in E the students were doing science: asking their own questions and doing things to find answers. For example, at one point in E a student asked, "Did it [the leaf] lose its colour?" and he tried to scratch a leaf to see if the green colour could be easily removed to make the leaf white. His implicit question was, “Why is the leaf white?” and he made a hypothesis which he tested in order to find out whether it was true. If he also saw that the teachers appreciated or validated his act, he may have been encouraged to continue to use such a scientific method throughout his life.

In D the students were expected to follow a step-by-step procedure which may be a [rather robotic] scientific method, but it is not clear whether they would internalise it, find it useful, or apply it in future. The objective was concentrated on using the teacher-given method to get to the content - and doing it to win a competition.

Concentrating on remembering ‘the body of knowledge’ and treating science as if it is just a list of static ‘facts’ is non-HDM. Concentrating on learning and doing science as a ***process***, and a process which cannot be separated from content, is more HDM.

We think both teachers and students need to question whether learning science as a process or remembering and understanding specific 'facts' (such as the names of trees) is necessary or desirable, and why. The answers will depend on our ways of looking at the world - our philosophies of science education, and our philosophies in general. Here we use the word 'ideology' to mean a philosophy or way of looking at the world. Accordingly, we all have 'ideologies', whether we are aware of them or not, and our actions are based on our ideologies - either consciously or unconsciously. We can become aware of our ideologies by examining our actions.

**Categories are fixed and static?**

Categorisation and naming are important aspects of the process of science, and they need to be (and usually are) components of science education. In examples C and D, each type of tree is a fixed, separate, and individual category. Each organism has one name, which is the name of the category it inherently belongs to. Identifying the names of organisms and remembering their names is an important science learning objective.

This contrasts with example E in which the students were allowed to suggest their own names for a tree. This is in line with an ideology in which it is recognised that something may fit in more than one category and have more than one name and different people may not agree on the name. Actually it is people who decide the categories and names of organisms, and people can change the names of organisms for particular reasons. Organisms are difficult to categorise because there is a large amount of variation between individual specimens as well as similarities between different species. These dialectically related similarities and variations are due to historical (evolutionary) interconnections between individuals and groups of individuals and their environments. Perhaps the students in E were beginning to see some interconnections when they compared the variegated and green bhendi trees and asked if they could have the same name.

**How is the content determined?**

Another difference between the examples we have given is that in C and D the content or topic (tree names) was determined by the teacher, textbooks, and school board syllabus, whereas in E the students took part in determining which particular topics they explored. Although the teacher brought them to the context - the tree - in the hope that they would discuss and explore its leaves, the students were free to explore any aspect of the tree (e.g. they also explored its thorns, flowers, and roots), and at times they even explored insects and other plants. The approach was thus more open-beginninged than C or D. The content was determined by the learners interacting with each other and their environment, and therefore local problems were central to learning. In A, some of the tree names were of trees that the students had never even seen before. The areas explored in E were more relevant because they arose through interconnections with students' own interests and experiences. The content was determined through a dialectical process in which the context (the environment) limited the topic to some extent, while the interactions between students with different backgrounds, experiences, and opinions expanded the exploration and made it more diverse and less individualistic, thus integrating it with an historical element. Out-of school knowledge/experience and interconnections were valued much more than in examples C or D. The material stuff - physical reality - was the basis of the learning/teaching in E, whereas the learning/teaching in C and D were based on ideas: names and the importance of knowing names, the material stuff was secondary.

The material stuff, if addressed at all in a less HDM approach, is treated as a mere reflection of an idea. As Kosambi points out, "The Sanskrit/Hindi word [पदार्थ] (padārtha) is literally "word-meaning", though it denotes "material object". So it is natural for the average Indian (sic) to assume, without a conscious effort on his (sic) part to the contrary, that objects are word-meanings, reflections of the word he makes up, of his ideas" (rather than abstract words being the reflections of the physical objects). (Kosambi, 1943)

**Dialectical argumentation and questioning arising out of conflict**

In example C, the teaching/learning was individual and separated from the natural environment. The trees which were the topic of study were not being observed directly. Apparently, learning was something that one does by sitting quietly, introspecting, and listening to or reading what an authority says.

However, if teaching/learning is based on an HDM ideology, teaching/learning is grounded in the local context, through interactions between peers and the material stuff, including the historical dimension. Learning may be through interactions, discussions, and arguments between students, between students and teachers, and between students and teachers and their environment. Although we did not see it in the examples here, interactions may even extend further, to include the local community and beyond.

In a scenario where content is not fixed and is determined by students by interacting with each other and the physical environment, the role of communication and interaction among peers becomes all the more important:

> "...dialectical argumentation requires, by definition, the examination and coordination of different perspectives. Participants are forced to acquire new information about the topic under consideration, since they are exposed to a multiplicity of ideas and encouraged to explore the validity of each of these ideas. This means that they have to consider objections to their personal theories and assumptions, to attempt to understand alternative positions." (Muller & Perret-Clermont, 2009)

As pointed out by Barnes (1992), for a curriculum to become meaningful, it has to be grounded in the communication and talk among students and teachers where they exchange meanings and can disagree with other's meanings.

Only when there are multiple perspectives, dialogical discourse can occur. Dialogical discourse plays an important role in knowledge construction. In a dialogical discourse people confront each other's ideas and confrontation of conflicting ideas is thought to be a "fair-play" (Muller & Perret-Clermont, 2009). Accordingly, accepting authoritarian or majority world view without confrontation does not contribute to knowledge construction.

Questioning is quite inherent to a dialogical discourse. Questioning arises due to the appearance of conflicts - or perplexity, as Dillon says (Dillon, 2004). The recognition of inherent inner conflicts in all processes is an essential component of HDM, and teaching/learning which is based on HDM must emphasize questioning. Since we are concerned about student learning, students must be the ones who are asking questions (as in example E), not just teachers (as in examples C and D).

## Some definitions and a tabular summary

### Idealism

Idealism is a worldview according to which ideas are held to be more basic than matter. In terms of "the relation of thinking to being", Idealism, as presented by Engels (2012 [1886] pp. 199-200) includes the worldviews of people who assert the "primacy of spirit to nature and, therefore… assume(d) world creation in some form or other". Citing Hegel as an idealist, Engels shows how for Hegel, "what we cognize in the real world is precisely (its) thought-content — that which makes the world a gradual realization of the absolute idea, which absolute idea has existed somewhere from eternity, independent of the world and before the world. But it is manifest without further proof that thought can know a content which is from the outset a thought-content."

In this paper, we address another aspect of idealism which manifests in the context of knowledge. Through an idealist worldview, ideas can be assumed to be known *a priori* i.e. derived from pure thought without engaging/struggling with material reality. For an idealist *a priorist*, as Engels (2011 [1885]) puts it, "principles, formal tenets (are) derived from thought and not from the external world, which are to be applied to nature and the realm of man, and to which therefore nature and man have to conform." We argue that an approach to science and mathematics education in which ideas are given primacy "which are to be applied to nature", is more idealist.

### Materialism

According to a materialist outlook nature and matter are taken to be more basic than ideas. From a materialist standpoint, the mind and consciousness are also derived from matter. In the context of knowledge, through a materialist worldview, as suggested by Engels (2011 [1885], p. 46), "The principles are not the starting-point of the investigation but its final result; they are not applied to nature and human history, but abstracted from them; it is not nature and the realm of humanity which conform to these principles..." We advocate the worldview that, questions and subsequent ideas arise from engaging with material reality, noticing contradictions, experiencing conflicts which necessitates conceptual change and the need for new ideas.

**Table 1: What is a more or less HDM ideology?** Our definition of the term, HDM (Historical Dialectical Materialism), is explained in this Table with reference to a few examples.

| **A less HDM ideology** | **A more HDM ideology** |
|---|---|
| **A is A and A is not non-A:** | **Processes inherently consist of inner conflicts: A is A and A is also non-A:** |
| A thing inherently belongs to a fixed, separate, and individual category, with one name and one definition. | People make and remake names and categories. A thing may have more than one name or category, and a name may refer to different things. |
| Each answer is either true or false - e.g. either a tree has thorns or does not. | An answer is inherently both/neither true and/or false – e.g. they are thorns since they look like thorns, but they are not thorns because they are too soft and flexible. |
| Things in one category are similar. | Things in one category are inherently similar and variable. |
| **Things are separate and individual:** | **Things are interconnected and interdependent:** |
| A thing may be separate and fixed - we can understand things by breaking them into separate pieces and studying each one individually (reductionism). | A thing (process) inherently consists of opposing forces - processes have to be understood in their relations and inter-connections with other processes |
| Division of subjects into separate domains (reductionism) and definition of topics, lessons, and syllabus by the teacher or another authority. | Interdisciplinary teaching - e.g. while exploring a tree, let students choose to investigate related questions: thorns, leaf colour, tree climbing, photosynthesis, the political economy of tree breeding, leaf poetry, or leaf shape. |
| Creation of 'school-school' separate from everyday life. | A better school is more similar to everyday life - the two overlap, both spatially and temporally |
| Focus on being private, individual. | Focus on the social/political/economic - the personal is political |
| Content and method are separate, and the most important thing to be taught is the content - the 'body of knowledge'. | Content and method are inseparable - concentrate on teaching through doing (the process of science) and some content will always be included - it is ok if all topics are not 'covered'. |
| Teaching/learning individually - not interacting with the environment - learning by sitting alone and thinking (or listening or reading). | Teaching/learning in interconnected groups - communities interacting with the environment - knowledge is grounded in the local context. |
| Content: what is to be learned is not determined by the learners but by the textbook, teacher, school board, etc - maybe foreign topics - may not be relevant to the students own lives - the medium of instruction may be a foreign language. | Content: what is to be learned is determined by the learners interacting with each other and the environment - local problems are central to learning - problems are more relevant because interconnections with students' own lives are stressed - their own languages are used. |

| **Things are static and fixed:** | **Everything changes (things are processes):** |
|---|---|
| The learning objective is to 'know' (remember) the science content - the static 'body of knowledge' (facts, concepts, laws, theories) - physical reality is secondary. | The learning objective is to creatively engage in the science ***process*** throughout one's life: to critically analyse physical reality, which is basic (concepts and words are based on real stuff). |
| In mathematics, the focus is on mathematical concepts. | In mathematics, the focus is on the process of mathematising that could lead to the construction of new concepts. |
| **Radical change is not possible:** | **Radical change is possible:** |
| Refusal to realise (or difficulty in realising) that gradual quantitative changes can lead to sudden, dramatic qualitative changes - which leads to difficulty in seeing examples of it in physical reality | Realise the possibility of and look for gradual quantitative changes leading to sudden, dramatic qualitative changes. |
| No one dares to question a well-established theory or conduct an experiment in order to test it, or question the definition of a particular type of tree or a definition of negative numbers. | Anything can be questioned - even students can do experiments in order to challenge well-established theories - everyone is encouraged to question the status-quo. |
| **Teaching/learning implications:** | **Teaching/learning implications:** |
| There is no universal scientific method, and traditional science education over-values experimental work and undervalues theoretical work.<br>There are many distinct ways of knowing besides through science, and any one way is as good as any other way. | A universal scientific method exists - and it consists of various numbers of aspects in various combinations (aspects include questioning, observing, testing, communicating, hypothesising, analysing, modelling, concluding, reasoning, etc.)<br>Everyone can use a scientific method to ask and answer questions, not just 'scientists' |
| Students need to be taught science concepts before they can do science. | Students learn science by doing science. In the process, they may come up with known or even novel concepts. |
| The teacher's role is to teach/transfer the content/concepts to students. | The teacher's role is to create the necessary material conditions for students to have the liberty to engage with the material world and facilitate their invention of concepts, as the need arises. They may also study concepts developed by others - and how and why they were developed. |
| First teach the idea, concept, theory, or theorem, and then think about a context where it can be applied. | First explore a material problem - and see how it can be modelled. Mathematical ideas were developed to model certain aspects of the material world. |
| Students do not study the political/economic interconnections and constraints on science (e.g. that the bottom line is the profit motive). | Students study the political/economic interconnections and constraints on science. |

| | |
|---|---|
| Mathematics offers precision, rigour, exactitude. | Mathematics is necessarily filled with questions and confusions: "All (Mathematical) models are false; some are useful." (G. E. P. Box)<br>Its unlikely that a particular model will describe a phenomenon with precision. |
| Mathematics out of context: 1+1=2<br>The colour of a triangle is not relevant: a triangle is an abstraction, colour is a physical characteristic. Mathematics is about abstractions. | 2 what? Mathematics in context: 2 jamun are different from 2 dragonflies.<br>A red triangle is different from a white triangle - the abstract idea of a triangle is based on physical triangularity, which is related to other physical characteristics such as colour. |
| Higher dimensional geometric objects (e.g. a 5D hypercube) is obviously non-tangible and thus derived purely from thought. | Geometric objects are developed by engaging with the material world and seeing whether it helps to extend a definition to higher or lower dimensions. |

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